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Linear Stability Analysis of a Non-Newtonian Liquid Sheet

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Gel propellant is a non-Newtonian visco-elastic liquid. The study on breakup of a gel-propellant sheet emanated from an injector has always been a fundamental task for a propulsion system with gelled propellants. A linear instability analysis method has been used to investigate the breakup of a visco-elastic liquid sheet under the combined influence of sinuous and varicose modes of disturbances at the liquid–gas interfaces. The disturbance wave growth rate of two disturbance modes has been worked out by solving the dispersion equation of a visco-elastic liquid sheet, which is obtained by combining the linear instability theory and the linear visco-elastic model. The linear visco-elastic model can describe the thixotropic behavior of the visco-elastic fluid. Moreover, the maximum growth rate and corresponding dominant wave numbers have been observed. The surface deformation growth rate curves, maximum growth rate curves, and dominant wave numbers curves have been plotted. Also, the breakup time of the liquid sheet has been computed by plotting the surface deformation curves based on the surface deformation equation on the liquid–gas interfaces. The time constant ratio, zero shear viscosity, surface tension, liquid sheet velocity, liquid sheet thickness, gas-to-liquid density ratio, and liquid density have been tested for their influence on the instability of the visco-elastic liquid sheet. Furthermore, the instabilities of Newtonian fluid and visco-elastic fluid are compared with each other. The results show that the visco-elastic liquid sheet is more unstable than the Newtonian liquid sheet. Similarly, the sinuous mode disturbance is more unstable than the varicose mode.

I. Introduction

GEL propellant is promising in future aerospace application because of its advantages in energetic performance and safety features. The gels offer the energy management of liquid propulsion and the storability and high density impulse of solid propulsion. Metallized gels have significantly improved density impulse in comparison to liquid propellants, depending on the metal additive and its loading. Gel is liquid propellant whose rheological properties have been altered by the addition of gellant and as a result its behavior resembles that of solid. From the view of fluid mechanics, gels behave as time dependent non-Newtonian visco-elastic fluids. In general, gel propellants exhibit shear-thinning (viscosity decreases with increasing applied shear stress), thixotropic (viscosity decreases in time at constant applied shear stress) behavior. The existence of yield stress and increased viscosity can prevent agglomeration, aggregation and separation of a metal solid phase from the fuel during storage. Consequently, these propellants are advantageous because of their long term storage capability. In addition, the visco-elastic properties reduce spill in cases of accidental leakage, hence increasing safety.

However, the rheological properties of gel cause the viscosity to increase, which makes the propellant much difficult to be atomized. It is known that in Newtonian fluids, large viscosity values produce coarse sprays and in non-Newtonian fluids shear and extensional viscosities can be several orders of magnitude larger. This results in reduced performance, and a longer combustion chamber is required (increased weight). To obtain high combustion efficiency, fine atomization is necessary. The atomization of non-Newtonian fluids is significantly different from the atomization of Newtonian liquids and very little is known about the influence of the rheological properties on the spray pattern of non-Newtonian fluids. Although some investigations have been performed to study the atomization characteristics of gel propellants applied to certain injector [1–4], the atomization mechanism of the gel propellants is yet to be studied.

It is well known that a circular jet and a thin sheet are two basic forms in which a liquid issues from an injector. Many experimental and theoretical investigations have been performed on the breakup mechanisms of liquid jets and liquid sheets, however, very little is known about the instability and breakup of non-Newtonian liquid sheets. Since the first studies by Savart, the breakup of flat sheets has been investigated by numerous other researchers [5–10]. For inviscid liquid sheets of uniform thickness in an inviscid gas environment the behavior of instability and breakup was analyzed by Squire [5] and Hagerty and Shea [6]. Their analytical results show that the surface tension always tends to damp out any protuberances, and the aerodynamic forces resulting from the interaction between the liquid sheet and the ambient gas are responsible for the instability of inviscid sheets. For Newtonian liquid sheets in an inviscid gaseous medium the characteristics of instability and breakup were analyzed by Dombrowski and Johns [7], and Li and Tankin [8]. Dombrowski and Johns [7] discussed the viscous effects on the instability of Newtonian sheets. Li and Tankin's [8] results showed that for Newtonian fluids the surface tension always opposes, while the relative motion between the sheet and gas favors the onset and development of instability.

The mechanisms of non-Newtonian liquid sheets are of both practical and theoretical interest. But up to now very little is known about the influence of gelling agents on the disintegration process and in contrast to ideal or Newtonian fluids, less investigation has been done [9–12]. From a theoretical point of view this problem is also of interest since a linear stability analysis of Newtonian liquid sheets is successful in predicting the characteristics of these sheets under certain conditions. It is therefore tempting to investigate whether a linear stability analysis would lead to similar results for non-Newtonian fluids. Chojnacki and Feikema [9] conducted a linear stability analysis of a flat two-dimensional non-Newtonian liquid sheet using the simple power-law equation (Ostwald-de Waele model), whereas the comparison of the theoretical results with

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experimental data shows an unsatisfactory correlation. One reason on this problem might be due to the high shear rates. For this purpose power-law models are unsuitable. As a result in the mentioned works it turns out that in the case of high Weber numbers non-Newtonian liquid sheets have higher growth rates for the both possible instability modes, i.e., varicose and sinuous modes compared with Newtonian fluids. This indicates that for large flow rates non-Newtonian liquid sheets are more unstable than liquid sheets with Newtonian attributes.

The results of the temporal stability analysis for a liquid sheet formed by two impinging jets show in the case of high viscous fluids as well as the liquids modeled by means of power-law in literature [10] an increasing growth rate, identify that a rise in viscosity tends to destabilize the fluid sheet flow. As a result of the conducted investigations the derived theoretical break up lengths and the critical wave lengths are in a good agreement compared with experimental data in a wide range of We-numbers.

The instability of non-Newtonian liquid sheets moving in an inviscid gaseous environment is investigated by Liu et al. [11] and Brenn et al. [12]. A linearized stability analysis shows that non-Newtonian liquid sheets have a higher growth rate than Newtonian liquid sheets for both varicose and sinuous disturbances, indicating that non-Newtonian liquid sheets are more unstable than Newtonian liquid sheets. It is discovered that the maximum growth rate of sinuous disturbances is always larger than that of varicose disturbances, while the dominant wave number of sinuous disturbances is always smaller than that of varicose disturbances. This indicates that sinuous disturbances always prevail over varicose disturbances for non-Newtonian liquid sheets. The same conclusion has also been gained by many investigators [5,11,13,14].

In this paper, a linear stability model is applied to a two-dimensional thin liquid film to investigate the breakup characteristics of the visco-elastic liquid sheet. The constitutive equation of linear visco-elastic fluids is used to describe the shear stress and its thixotropic behavior, rather than power-law model which cannot describe the thixotropic behavior of the visco-elastic liquid.

II. Breakup Model for Visco-Elastic Liquid Sheet

A. Linear Visco-Elastic Model

As shown in Fig. 1, a two-dimensional, incompressible visco-elastic liquid sheet of thickness $2h$ moving with velocity $U(U_x, U_y)$ through a quiescent, inviscid, incompressible gas medium is considered. The liquid and gas have densities of ρ_l and ρ_g, respectively, and the surface tension of liquid is σ. μ_l denotes the viscosity of the visco-elastic fluid.

The study of this paper is based on a linear visco-elastic model, which is expressed as follows:

$$\tau_{ij} + \lambda_1 \frac{\partial \tau_{ij}}{\partial t} = -\mu\left(d_{ij} + \lambda_2 \frac{\partial d_{ij}}{\partial t}\right) \tag{1}$$

Where τ_{ij} is the shear stress, d_{ij} is the (i, j) component of the rate of deformation tensor, μ is the zero shear viscosity, λ_1 and λ_2 denotes the stress relaxation time and deformation retardation time,

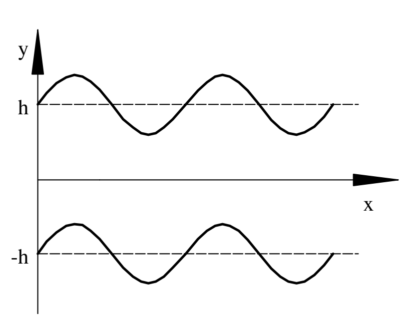


Fig. 1 Schematic of liquid sheet.

respectively. This model is a somewhat more versatile model with three constants $(\mu, \lambda_1, \lambda_2)$ put forward by Jeffreys in 1929 [15]. The ratio of λ_2/λ_1 is defined as time constant ratio denoted by $\bar{\lambda}$. For gel $\lambda_2 < \lambda_1$ as gel is shear-thinning fluid [16].

According to literature [17], assuming that variables U_x, U_y can be written in the general form $f(y, x, t) = F(y)e^{ikx+\alpha t}$, which is called the temporally growing and spatially harmonic wave form [18] or infinite extent-initial value problem [19] and has also been used by Gordon et al. [20] and Sadik and Zimmels [21], the following equation can be established:

$$(1 + \alpha\lambda_1)T_{yx} = -\mu(1 + \alpha\lambda_2)\left(ikU_y + \frac{\mathrm{d}U_x}{\mathrm{d}y}\right) \tag{2}$$

Where $k = 2\pi/\lambda$ is the wave number, and α is the growth rate of a perturbation. The most unstable disturbance has the largest value of α, denoted by $\alpha_{\max}$ in the present work, and is assumed to be responsible for breakup. The resulting ligament size is related to the maximum unstable wavelength $\lambda_s = 2\pi/k_d$ where k_d is the wave number corresponding to the maximum growth rate $\alpha_{\max}$.

Equation (2) reduces to

$$T_{yx} = -\mu_l\left(ikU_y + \frac{\mathrm{d}U_x}{\mathrm{d}y}\right) \tag{3}$$

where

$$\mu_l = \mu\frac{1 + \alpha\lambda_2}{1 + \alpha\lambda_1} \tag{4}$$

The shear stress of Newtonian fluid in the same form can be expressed as

$$\tau_{yx} = -\mu\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) \tag{5}$$

Considering the general form $f(y, x, t) = F(y)e^{ikx+\alpha t}$, Eq. (5) should be rewritten as

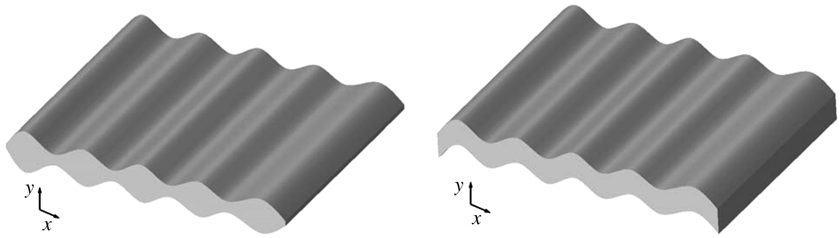
$$T_{yx} = -\mu\left[ikU_y(y) + \frac{\mathrm{d}U_x(y)}{\mathrm{d}y}\right] \tag{6}$$

By comparing Eqs. (3) and (6), it is obviously to find that the dynamic equations for this visco-elastic model are identical to those for the Newtonian fluid, with μ replaced everywhere by μ_l. The boundary conditions would also differ only by changing μ into μ_l. Accordingly, it is acceptable to adopt the dispersion relation of Newtonian fluid for visco-elastic fluid, just replacing μ with μ_l.

B. Dispersion Relation of Visco-Elastic Fluid

If a protuberance is produced on the interface owing to any disturbance, forces acting on the interfaces develop. The surface tension always tends to restore the interface back into its original equilibrium position, while the disturbance generally enhances the degree of instability, i.e., increase the amplitude of the disturbance. A relative velocity between the liquid and gas promotes the growth of disturbances until the liquid sheet disintegrates into fragments, which rapidly contract into unstable ligaments under the effect of surface tension. Finally, the ligaments are broken into a multitude of droplets. In this paper there are two types of disturbance for liquid sheet. When the phase contrast of distortion on two interfaces is 180°, it is called varicose disturbance wave (Fig. 2a); while the distortion on two liquid-gas interfaces is same-phase, it is called sinuous disturbance wave (Fig. 2b).

As analyzed by Senecal et al. [22], the dispersion relation of a Newtonian liquid sheet, which is subjected to a spectrum of infinitesimal disturbances of the form $\eta = \Re[\eta_0 \exp(ikx + \omega t)]$, is given by



a) Varicose disturbance wave **b) Sinuous disturbance wave**

Fig. 2 Mode of disturbance on the liquid sheet.

$$\begin{cases}\omega_{rs} = -\frac{2\mu k^2 \tanh(kh)}{\rho_l(\tanh(kh)+Q)} \\ \quad + \frac{\sqrt{4\mu^2 k^4 \tanh^2(kh)/\rho_l^2 - Q^2U^2k^2 - [\tanh(kh)+Q](-QU^2k^2+\sigma k^3/\rho_l)}}{\tanh(kh)+Q} \\ \omega_{is} = -\frac{iQkU}{\tanh(kh)+Q}\end{cases} \tag{7}$$

for the sinuous mode, where Q is the ratio of the gas density to liquid density ρ_g/ρ_l. Furthermore, the corresponding dispersion relation for the varicose mode can be obtained by replacing hyperbolic tangent $\tanh(kh)$ in Eq. (7) with hyperbolic cotangent $\coth(kh)$, namely,

$$\begin{cases}\omega_{rv} = -\frac{2\mu k^2 \coth(kh)}{\rho_l(\coth(kh)+Q)} \\ \quad + \frac{\sqrt{4\mu^2 k^4 \coth^2(kh)/\rho_l^2 - Q^2U^2k^2 - [\coth(kh)+Q](-QU^2k^2+\sigma k^3/\rho_l)}}{\coth(kh)+Q} \\ \omega_{iv} = -\frac{iQkU}{\coth(kh)+Q}\end{cases} \tag{8}$$

As stated before, the dispersion relation of the visco-elastic liquid sheet can be obtained by replacing μ with μ_1, and $\mu_1 = \mu\frac{1+\omega_r\lambda_2}{1+\omega_r\lambda_1}$ (where ω_r has the same meaning as α in Eq. (4), indicating the disturbance growth rate). When the stress relaxation time and deformation retardation time equal zero, the dispersion relation of the visco-elastic liquid sheet is transformed into that of Newtonian liquid sheet.

C. Breakup Time Calculation

On the liquid–gas interfaces, disturbances are present in the form of sinuous and varicose waves with a phase angle θ between them. Therefore, the initial condition for the surface deformation can be written as

$$\eta_j(x,0) = \eta_0[\varepsilon_1 \cos(kx) + (-1)^{j+1}\varepsilon_2 \cos(kx+\theta)] \tag{9}$$

Where $j = 1, 2$ refers to the outside and inside interface, respectively; ε_1 and ε_2 are the weighting factors denoting a pure sinuous wave for $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$, and a pure varicose wave for $\varepsilon_1 = 0$ and $\varepsilon_2 = 1$.

Then the final surface deformation equation can be expressed as follows [23]:

$$\eta_j(x,t) = \eta_0[\varepsilon_1 \cosh(\omega_{is}t)\cos(\omega_{rs}t + k_s x) + (-1)^{j+1}\varepsilon_2 \cosh(\omega_{iv}t)\cos(\omega_{rv}t + k_v x + \theta)] \tag{10}$$

k_s and k_v denote the wave numbers of the sinuous and varicose modes. To get the nondimensional breakup time, the growth rate, angular frequency and wave number should be nondimensional ones. The characteristic time for dimensionless is $[T] = \sqrt{\rho_l(2h_0)^3/\sigma}$, and the characteristic length is $[L] = 2h_0$. In other words, the growth rate, angular frequency should be multiplied by $\sqrt{\rho_l(2h_0)^3/\sigma}$ so as to become nondimensional ones, while the wave number by $2h_0$. Where, $\rho_l = 910\ \mathrm{Kg/m^3}$, $2h_0 = 0.001$ m, $\sigma = 0.07$ N/m.

Plotting the liquid sheet surface wave deformation at different times based on the final surface deformation equation on the liquid–gas interfaces can estimate the breakup time, at which the liquid sheet is thin enough and the amplitude is large enough. Then, it is easy to conclude the effects of different parameters on the breakup time of the liquid sheet by estimating adequate values in different cases.

III. Results and Discussion

A. Disturbance Growth Rate

1. Disturbance Growth Rate of Two Modes

As we know, the liquid sheet tends to be unstable when its disturbance growth rate is positive, which can be obtained through solving the dispersion relations numerically. Figure 3 shows the variation of the disturbance growth rate ω_r with the wave number k.

Evidently, with the increase of the wave number the disturbance growth rates increase at first, and then decrease for both modes. Consequentially, the growth rate of varicose mode should be negative when the wave number exceeds some critical value, which is 7000 probably for the visco-elastic fluid studied in this paper. The wave number range of 0–7000 can be defined as instability range, at which the disturbance growth rate curve lies above zero. However, the growth rate of sinuous mode keeps positive for all the wave number, but tends towards zero with the increase of the wave number. Thus, the instability range of sinuous mode can be regarded as $0 - \infty$. Therefore, it is acceptably to conclude that the sinuous mode disturbance is unstable for the disturbance whose wave number ranges from zero to infinity.

Moreover, the growth rate of sinuous mode is far greater than that of varicose mode, while the opposite is true for the dominant wave number, which indicates that sinuous disturbances always prevail over varicose disturbances for visco-elastic liquid sheets. And the disturbance wave in the breakup process would present a long wave because the dominant wave number of the sinuous disturbance is much less than that of the varicose disturbance.

Figure 3 also exhibits the comparison of visco-elastic fluid and Newtonian fluid. For sinuous mode the growth rate of visco-elastic fluid is greater than that of Newtonian fluid, while they are almost the same for varicose mode. Therefore, it is concluded that at the same condition (including the identical μ) a sheet of such a visco-elastic fluid is more unstable than a Newtonian liquid sheet against small disturbances. Similar results were reported by Madlener et al. [2] and Liu et al. [11] in their studies of instability of non-Newtonian liquid sheet.

2. Effect of the Time Constant Ratio

To investigate the effect of the time constant ratio $\bar{\lambda}$ on the disturbance growth rate, six values are chosen for calculation with the other parameters invariable. Although for gel $\lambda_2 < \lambda_1$, some more conditions that $\lambda_2 \geq \lambda_1$ are calculated for the purpose of comparison. The result is shown in Fig. 4. For the purpose of contrasting the visco-elastic fluid and Newtonian fluid, a curve of Newtonian fluid is also plotted. The variation of maximum growth rate and dominant wave number with time constant ratio are also plotted in Fig. 4.

In Fig. 4, with increase of the time constant ratio the maximum growth rate and the dominant wave number decrease for sinuous mode, while for varicose mode they seem to be unalterable. In some lectures, a nondimensional number $EI = \lambda_1\mu/\rho_l h^2$ is defined to describe the effect of elasticity. Therefore, in some sense λ_1 can represent the effect of elasticity, and the effect of elasticity is increased as the time constant ratio $\bar{\lambda}$ decreases. Therefore, from Fig. 4 we can get the conclusion that the liquid elasticity effects always tend to increase the wave growth rate in non-Newtonian

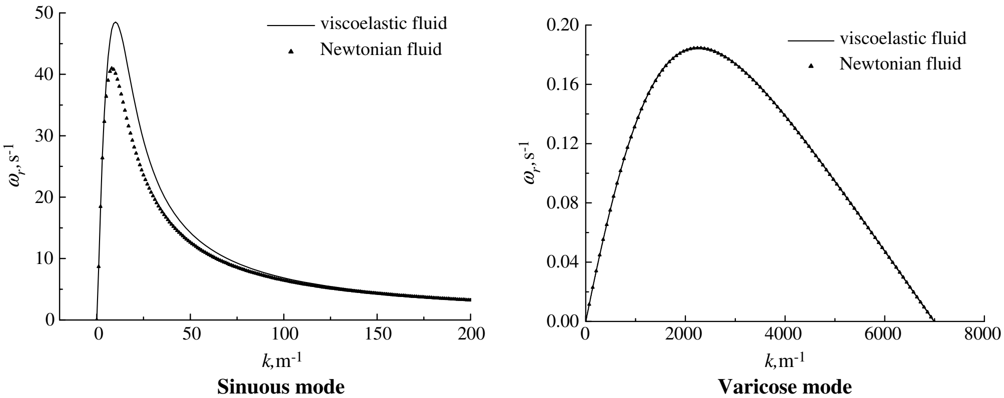


Fig. 3 Variation of growth rate with wave number. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

liquid sheets for sinuous mode disturbance. However, Fig. 4 also indicates that the growth rates of sinuous mode with different time constant ratios keep almost identical when the wave numbers are less than 5 or more than 150 (in this paper's precision range). It can be concluded that when the wave number is between 5 and 150, the time constant ratio may effects the disturbance growth rate distinctly for sinuous mode, i.e., the greater time constant ratio $\bar{\lambda}$ will slower the breakup of the visco-elastic fluid sheet. In addition, the instability

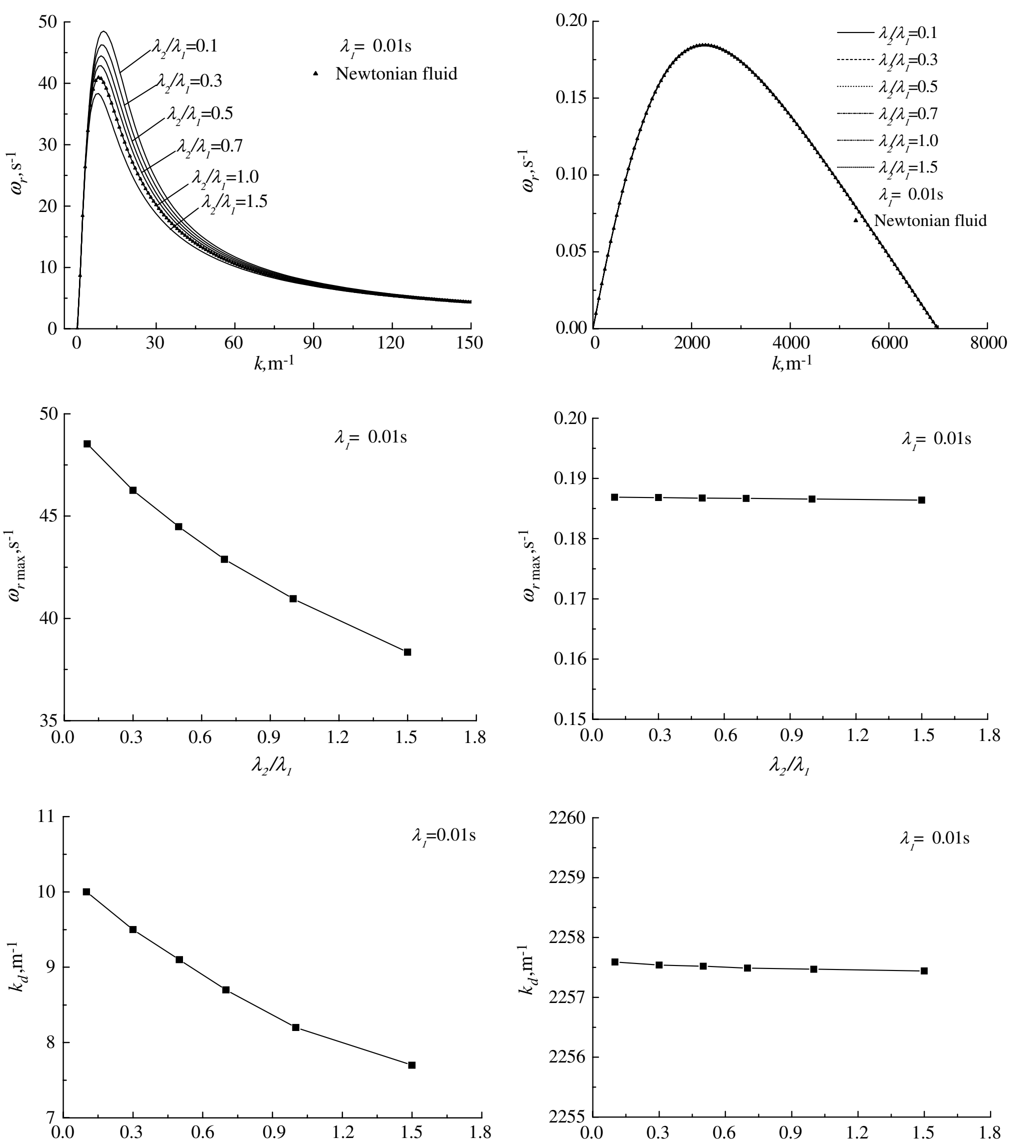


Fig. 4 Effect of time constant ratio on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode) (for visco-elastic fluid $\lambda_1 = 0.01$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

ranges of both sinuous and varicose modes hardly change with the time constant ratio.

Moreover, the curve denoting the Newtonian fluid coincides with that denoting the fluid whose $\bar{\lambda}$ equals 1. It can be explained logically: when $\bar{\lambda} = 1$, the viscosity $\mu_l = \mu \frac{1+\omega_r \lambda_2}{1+\omega_r \lambda_1} = \mu$, which means that the fluid is Newtonian fluid.

Comparing the sinuous mode with the varicose mode, it is distinctly that the growth rate of sinuous mode is greater than that of varicose mode, which has been analyzed in section I.

3. *Effect of the Zero Shear Viscosity*

Figure 5 explores the effects of the zero shear viscosity on the evolution of instabilities of visco-elastic liquid sheets, by increasing the zero shear viscosity from 180 to 1090 $\mathrm{Pa \cdot s}$ and keeping the other parameters constant. The results are shown for sinuous and varicose modes disturbances, respectively. The growth rate decreases with the increase of the zero shear viscosity for sinuous mode. Also, this trend is observed clearly for varicose mode. Moreover, it is observed that as the liquid zero shear viscosity increases, for the wave numbers more than a critical value (400 for Fig. 5) the values of the disturbance growth rates remain almost identical in a appropriate precision range for sinuous mode, while for the wave numbers more than a value (7000 for Fig. 5) the growth rates become negatives for varicose mode, i.e., the varicose mode disturbance keeps stable for the wave number more than 7000.

Therefore, it is concluded that with the wave number under some value (for example, 400 for sinuous mode and 7000 for varicose mode in this paper's calculation), the liquid zero shear viscosity effects always tend to decrease the wave growth rate in visco-elastic liquid sheets for both sinuous and varicose modes disturbances. However, the instability ranges do not change with the zero shear viscosity.

Furthermore, in Fig. 5 the curve denoting Newtonian fluid (viscosity $\mu_l = 360\ \mathrm{Pa \cdot s}$) lies below that denoting visco-elastic fluid with the zero shear viscosity $\mu = 360\ \mathrm{Pa \cdot s}$ for sinuous mode, while the two curves coincide with each other for varicose mode. Thus, it further confirms the conclusion of section I that at the same condition a sheet of such a visco-elastic fluid is more unstable than a Newtonian liquid sheet against small disturbances.

4. *Effect of the Surface Tension*

Figure 6 exhibits the effects of the surface tension on the maximum growth rate and the dominant wave number at the conditions given before except the surface tension σ varied stepwise from 0.02 to 0.12 for sinuous and varicose modes disturbances, respectively. Unlike the

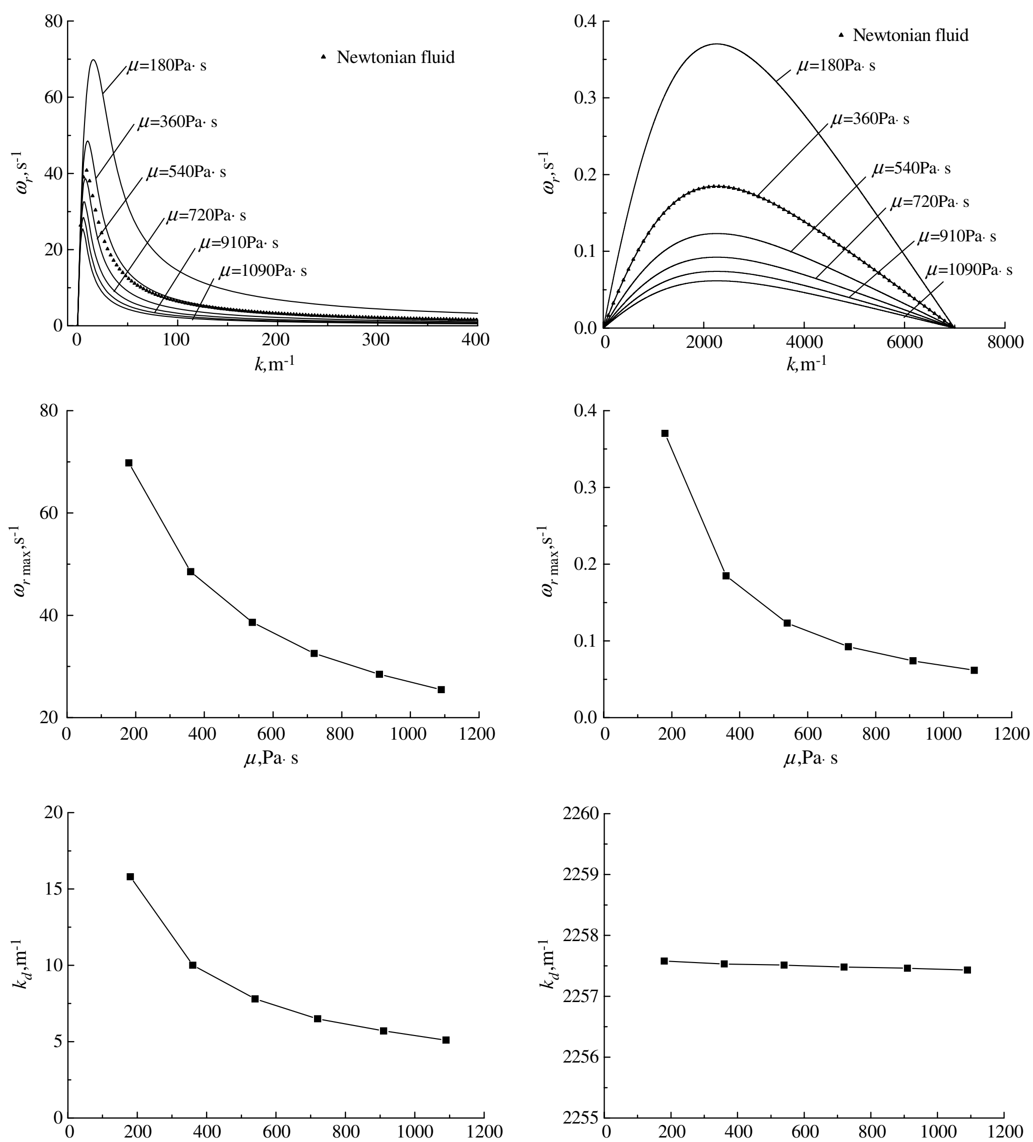


Fig. 5 Effect of zero shear viscosity on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910\ \mathrm{kg/m^3}$, $\rho_g = 1.225\ \mathrm{kg/m^3}$ and $\lambda_1 = \lambda_2 = 0$ s, $\mu = 360\ \mathrm{Pa \cdot s}$ for Newtonian fluid.

results aforementioned, the surface tension has nothing to do with the sinuous mode disturbance nearly, while the varicose mode disturbance growth rate changes acutely with the variation of the surface tension. It can attribute to the fact that the varicose mode brings the two interfaces close enough and the effect of surface tension gets greater. Then it can be concluded that the effects of surface tension on the varicose mode disturbance is greater than it on the sinuous mode disturbance. It is also seen from Fig. 6 that with increase of the surface tension both the maximum growth rate and the dominant wave number decrease for varicose mode disturbance, that is, the surface tension can resist the occurrence and development of instability in visco-elastic liquid sheets, or tend to smooth out disturbances on the interface between the liquid and the gas for varicose mode.

Furthermore, it is different from the sections aforementioned that the instability range diminishes as the surface tension increases for varicose mode, which indicates that the surface tension would make the liquid sheet stable and the breakup of liquid sheet go towards long wave mode (because the dominate wave number decreases).

However, the growth rate of sinuous mode is greater than that of varicose mode. So, the sinuous mode is still more unstable than varicose mode. In addition, the curve denoting Newtonian fluid (surface tension $\sigma = 0.07$ N/m) lies between the two curves denoting visco-elastic fluid whose $\sigma = 0.06$ and $\sigma = 0.08$ N/m respectively for varicose mode, and lies below all the curves for sinuous mode. Therefore, it confirms the conclusion that with all the parameters identical a visco-elastic liquid sheet is more unstable than a Newtonian liquid sheet against small disturbances for sinuous mode, but for varicose mode they are as stable as each other.

5. *Effect of the Liquid Sheet Velocity*

The effect of the liquid sheet velocity on the wave growth rate is examined in Fig. 7 for sinuous and varicose disturbances, respectively. From inspection of these figures, it is clear that, when the liquid sheet velocity is increased, both the growth rates and the dominant wave numbers of sinuous and varicose modes increase drastically. Therefore, it is concluded that the high liquid sheet velocity enhances the instability of visco-elastic liquid sheets for both sinuous and varicose modes disturbances. It can attribute to a higher relative velocity between liquid and gas phase enhances the aerodynamic effect and accelerates the breakup of visco-elastic liquid sheet. It is also obvious that sinuous disturbances prevail over varicose disturbances for visco-elastic liquid sheets as the growth rate of sinuous mode is much greater than that of varicose mode.

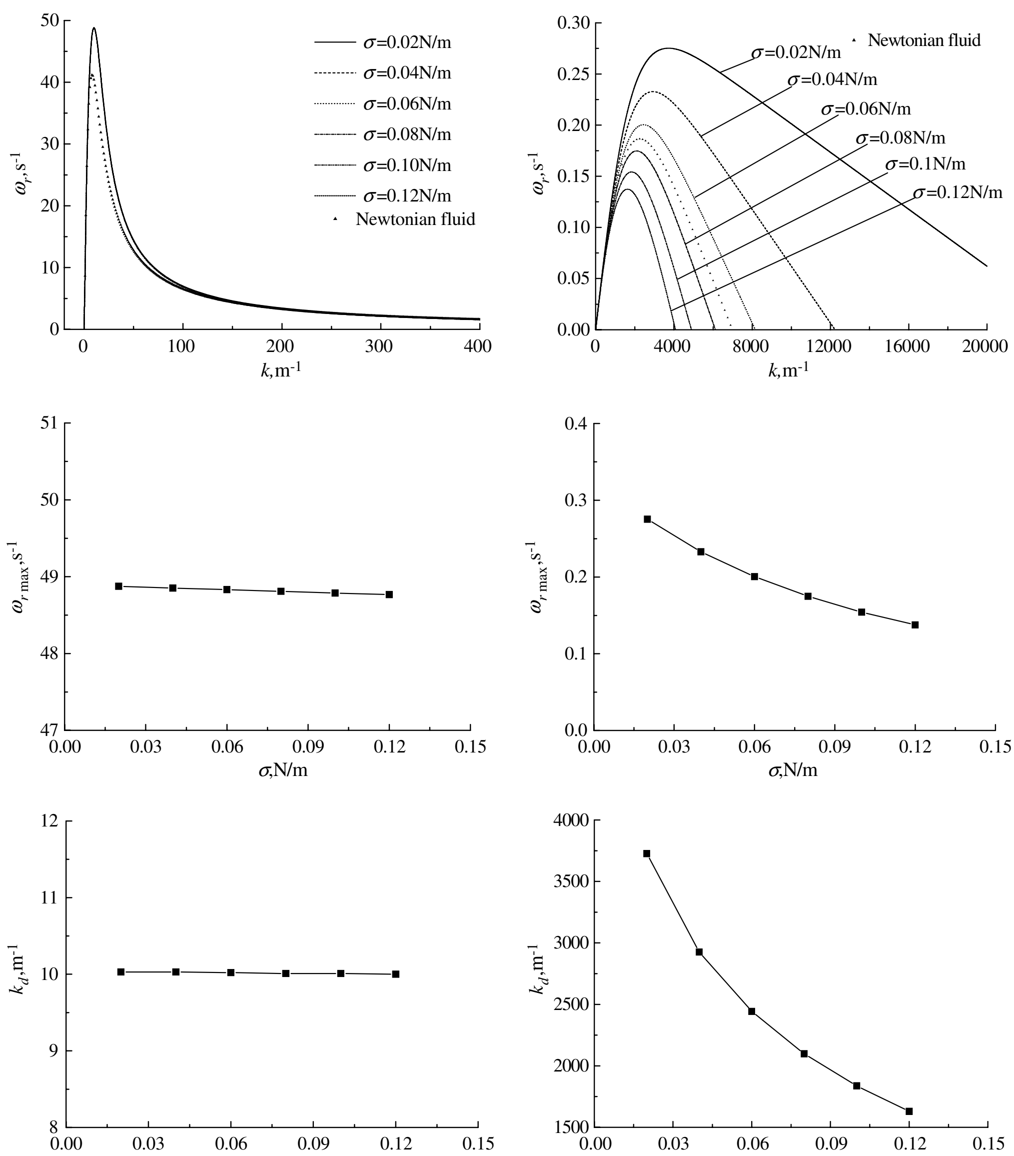


Fig. 6 Effect of surface tension on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s, $\sigma = 0.07$ N/m for Newtonian fluid.

Furthermore, the maximum growth rate curves reveal that the maximum growth rate increases exponentially with the increase of liquid sheet velocity almost, which indicates that the liquid sheet velocity can effect the disturbance growth rate sharply. Contrary to the effect of the surface tension, with the increase of liquid sheet velocity the instability range of varicose mode increases while that of sinuous mode remains $0-\infty$, which further confirms the conclusion that the liquid sheet velocity can accelerate the breakup of visco-elastic liquid sheet.

Similar to all the cases aforementioned, the growth rate curve of Newtonian liquid sheet coincides with the curve of visco-elastic liquid sheet with same parameter for varicose mode disturbance and lies below that denoting visco-elastic liquid sheet with same parameter for sinuous mode disturbance.

6. *Effect of the Liquid Sheet Thickness*

Figure 8 shows the effect of the liquid sheet thickness on the growth rate and wave number for sinuous and varicose modes disturbance, respectively, where the thickness varies from 0.0002 to 0.0012 m, and the other parameters remain unaltered. From inspection of these figures, it is obvious that both the maximum growth rate and the dominant wave number of sinuous and varicose modes disturbances change substantially with the variation of the liquid sheet thickness. It is amazing to find out that with increase of the thickness the variety trends of sinuous and varicose disturbance are contrary to each other, that is, the maximum growth rate decreases with the increase of the thickness for sinuous mode disturbance, while the opposite is true for varicose mode disturbance. However, the dominant wave number decreases with the increase of liquid sheet thickness for both sinuous and varicose modes disturbances. Based on these phenomena, it is concluded that increasing liquid sheet thickness results in accelerating the varicose mode disturbance but restraining the sinuous mode disturbance.

Moreover, different from the effects of surface tension and velocity, the variety of liquid sheet thickness does not change the instability range of varicose mode disturbance, which is similar to the effects of time constant ratio and zero shear viscosity. Thus, the thickness affects the varicose disturbance on the liquid sheet surface only by enhancing the growth rate in the instability range.

From Fig. 8 many other trends can also be found similar with the sections aforementioned, that is, the growth rate of sinuous mode is much greater than that of varicose mode; and the growth rate curve of Newtonian liquid sheet lies below that of sinuous mode but almost

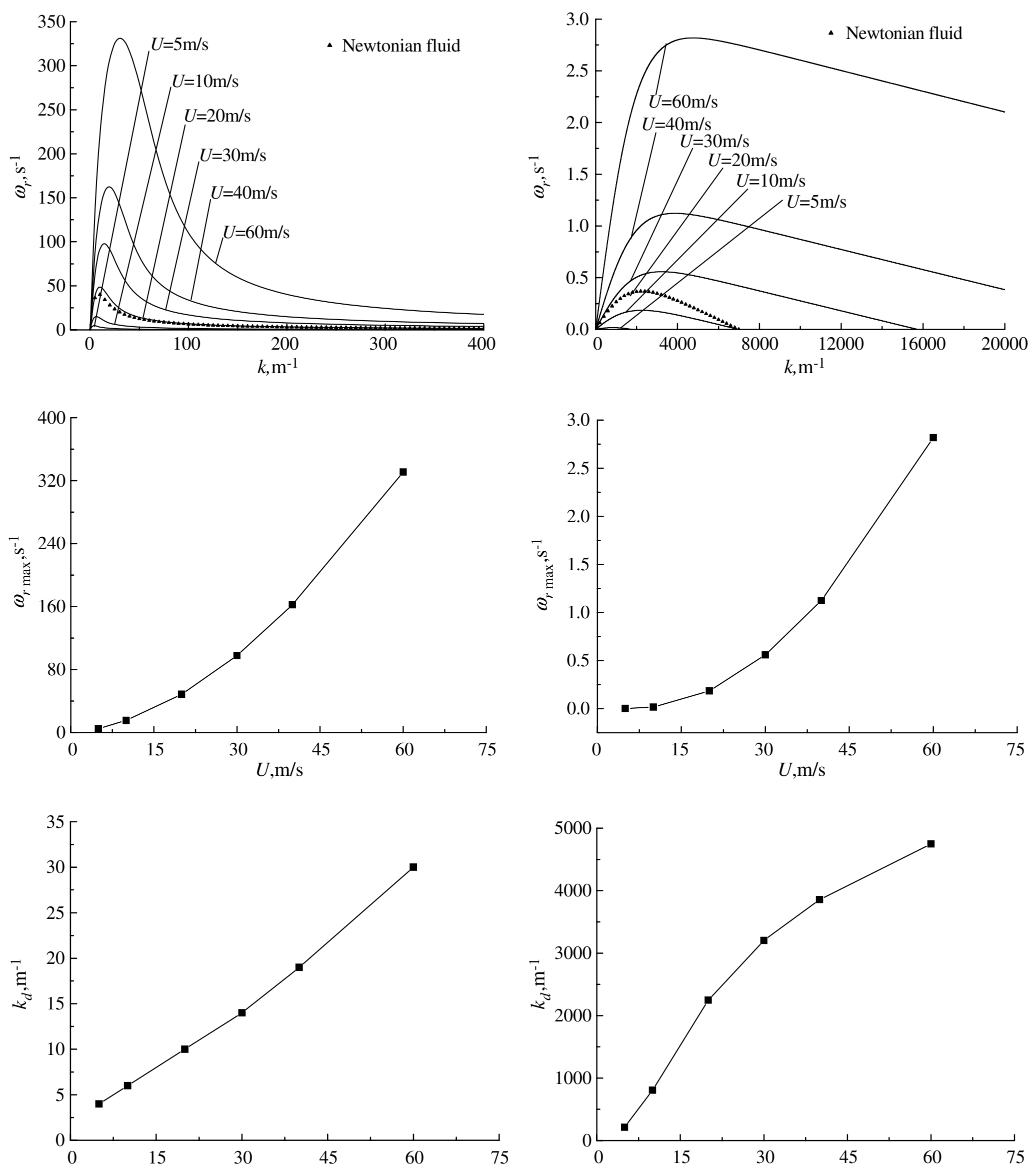


Fig. 7 Effect of liquid sheet velocity on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s, $U = 20$ m/s for Newtonian fluid.

coincides with that of varicose mode when the parameters are the same.

7. Effect of the Gas-to-Liquid Density Ratio

Figure 9 gives the effects of the density ratio of gas to liquid on the growth rate and the dominant wave number of sinuous and varicose modes disturbances by increasing Q from 0.0005 to 0.03 (ρ_g from 0.5 to 30 $\mathrm{kg/m^3}$) and keeping the other parameters constant. From the observation of these figures, it is clear that, when the density ratio of gas to liquid is increased, both the growth rates and the dominant wave numbers of sinuous and varicose modes disturbances increase drastically, which means the gas-to-liquid density ratio Q can accelerate the disturbance of both sinuous and varicose modes. Furthermore, the gas-to-liquid density ratio is increased by changing the gas density but keeping the liquid density constant. So it is reasonable to owe the effect of Q to the gas density, in other words, the high ambient gas density enhances the instability of visco-elastic liquid sheets for both sinuous and varicose modes disturbances.

Moreover, similar to the effects of liquid sheet velocity, the increase of the gas-to-liquid density ratio will lead to the increase of the instability range for varicose mode disturbance, confirming that the increase of the gas-to-liquid density ratio Q can make the visco-elastic liquid sheets more unstable.

Similarly, the comparison of Newtonian fluid and visco-elastic fluid is found to be similar to the sections aforementioned also; the growth rate of sinuous mode disturbance is much great than that of varicose mode. This result is also been gained by many investigators [5,11,13,14]. However, Sirignano and Mehring's review paper [24] stated (citing Li and Tankin [8] and Rangel and Sirignano [25]) that at low gas-to-liquid density ratios, the growth rate of the sinuous waves is larger than that of the varicose waves, in agreement with our results; while at higher gas-to-liquid density ratios (higher than a critical value, 0.25 in literature 24), it is shown that the varicose waves have a higher growth rate. But the regime of higher density ratio is rarely reached in practice, for the density ratio of gas to liquid encountered in practice is much smaller than the critical value. And our calculations are targeted at the gas-to-liquid density ratio ranging from 0.0005 to 0.03. After comparison and thorough consideration we draw the conclusion that it may attribute to the difference of the non-Newtonian and Newtonian liquid or the different Weber numbers. The essential reason needs more research.

8. Effect of the Liquid Density

Considering that the liquid density may affect the growth rate with the gas-to-liquid density ratio kept invariable, some more calculations are done with the liquid density varied from 850 to

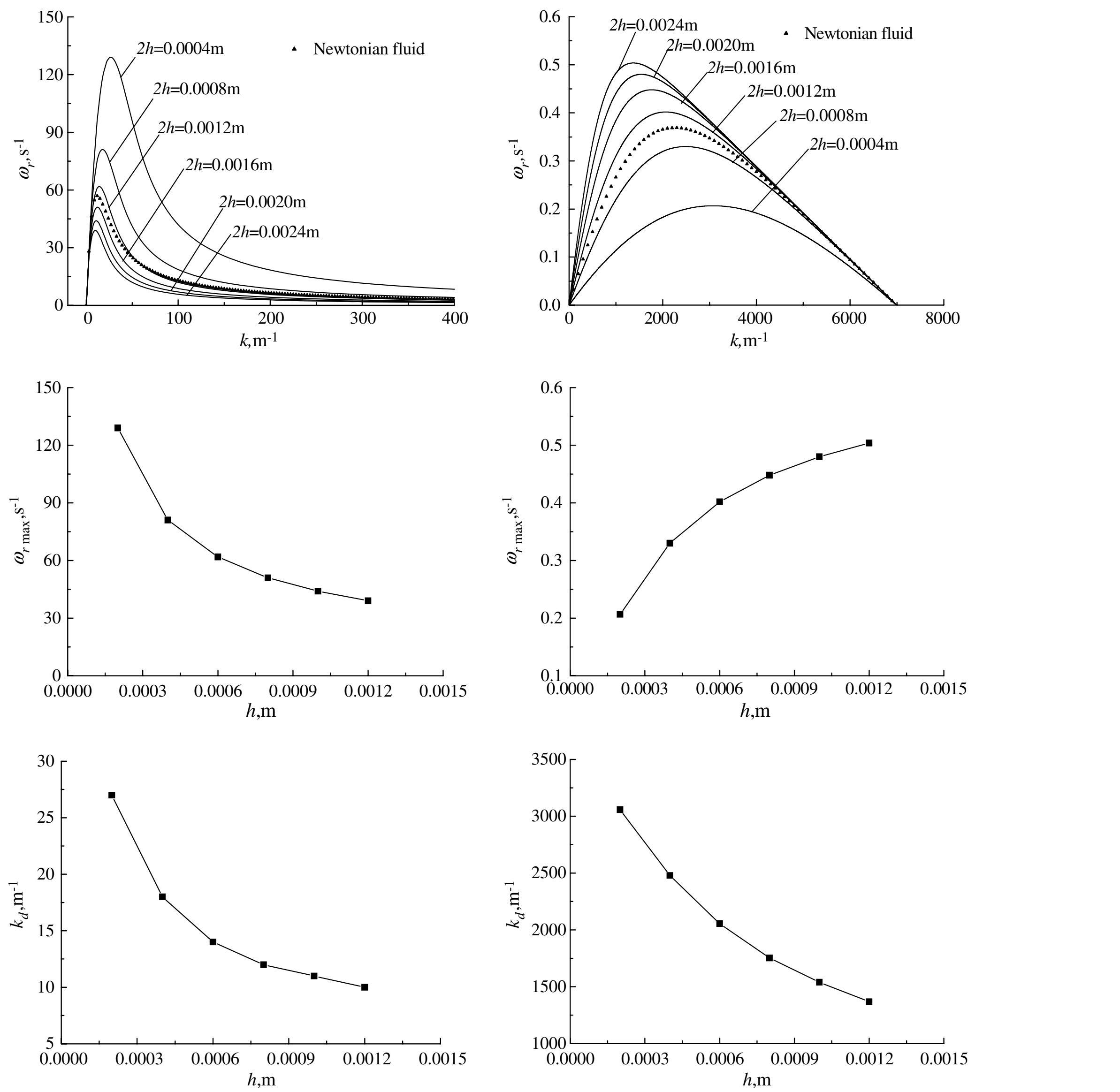


Fig. 8 Effect of liquid sheet thickness on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s, $2h = 0.001$m for Newtonian fluid.

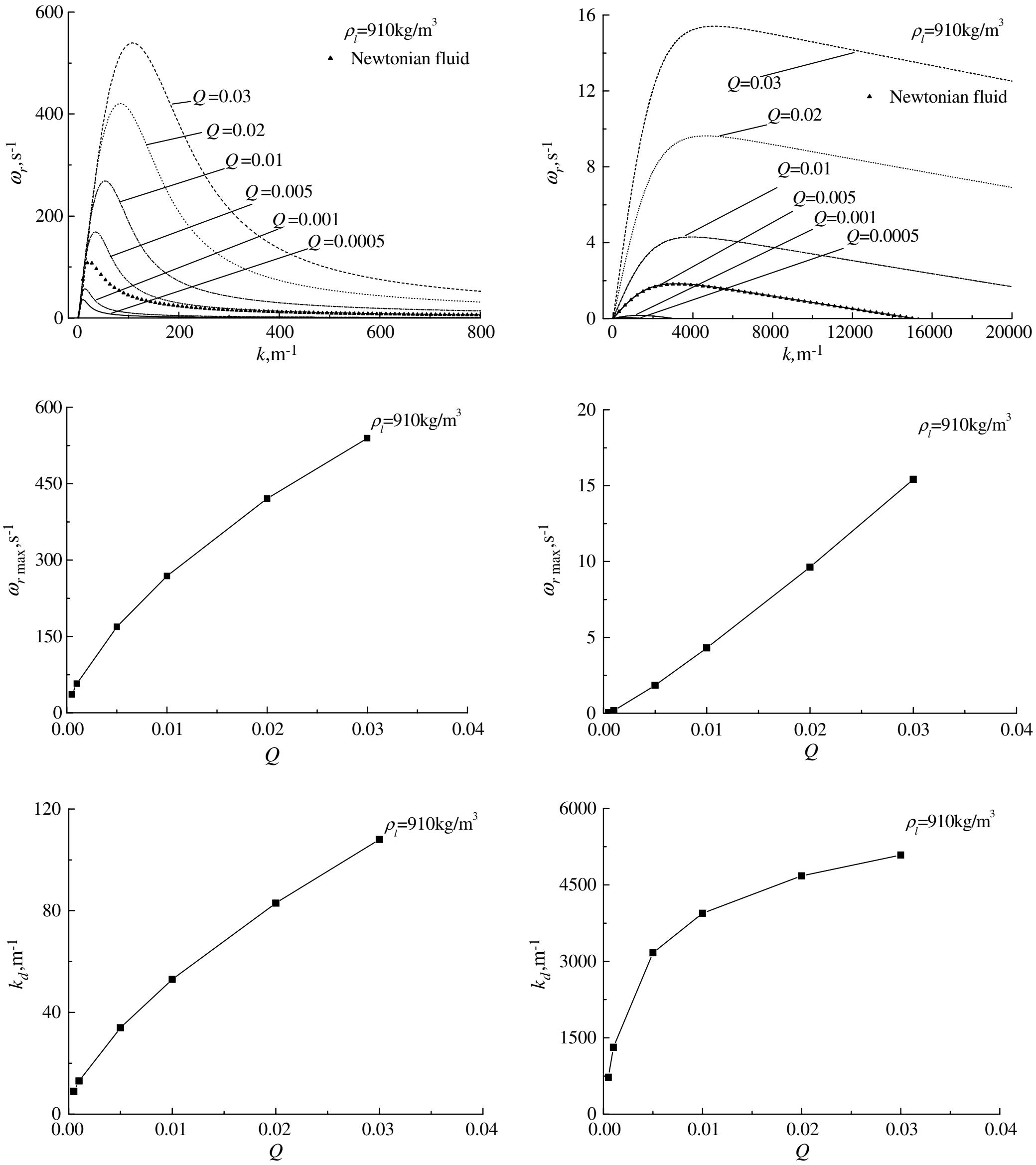


Fig. 9 Effect of gas-to-liquid density ratio on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $Q = \rho_g/\rho_l$ and $\lambda_1 = \lambda_2 = 0$ s, $Q = 0.005$ for Newtonian fluid.

1100 kg/m^3 and the gas-to-liquid density ratio Q kept 0.0013. The results are shown in Fig. 10. However, these curves reveal that when the gas-to-liquid density ratio is constant, the liquid density can hardly affect the growth rate of sinuous mode disturbance and affect that of varicose mode slightly, i.e., with the increase of liquid density the growth rate increases for varicose mode disturbance. Therefore, it is discovered that the liquid density reacts on the liquid sheet instability little when the gas-to-liquid density ratio is determined for sinuous mode disturbance, while it can enhance the instability to a certain extent for varicose mode disturbance.

Moreover, differing from liquid density it is absolutely impossible for the gas density to react on the liquid sheet instability when Q is constant, which can be proved by the dispersion relation directly.

B. Breakup Time of Liquid Sheet

1. *Effect of the Time Constant Ratio*

Figure 11 exhibits the effect of the time constant ratio on the breakup time for both sinuous and varicose modes. As for Newtonian liquid sheet the stress relaxation time λ_1 and deformation retardation time λ_2 equal zero both or $\lambda_1 = \lambda_2$, only one case can be considered as Newtonian liquid, namely, the time constant ratio equals 1, which is denoted by a circle. From inspection of these two curves, it is seen that with the increase of the time constant ratio the breakup time increase for sinuous mode disturbance. In other words, the time constant ratio can decelerates the breakup of the liquid sheet for sinuous mode.

Comparing Figs. 11 and 4 confirms the trend that with the increase of the time constant ratio the visco-elastic liquid sheet tends to be difficult to breakup for sinuous mode disturbance, while for varicose mode the breakup time seems to be stable.

Furthermore, these two curves also reveal that when the time constant ratio is less than 1, the breakup time of the visco-elastic liquid sheet is shorter than that of Newtonian liquid sheet, whose viscosity equals the zero shear viscosity of the visco-elastic liquid, while the opposite is true when the time constant ratio exceeds 1.

2. *Effect of the Zero Shear Viscosity (Viscosity)*

Figure 12 shows the variation of the breakup time with the zero shear viscosity (viscosity for Newtonian liquid), which indicates that the breakup time increases with the increase of the viscosity for both modes. Thus, it can be concluded that the zero shear viscosity (viscosity for Newtonian liquid) decelerates the breakup of the liquid sheet, i.e., a higher viscosity leads to a longer breakup time.

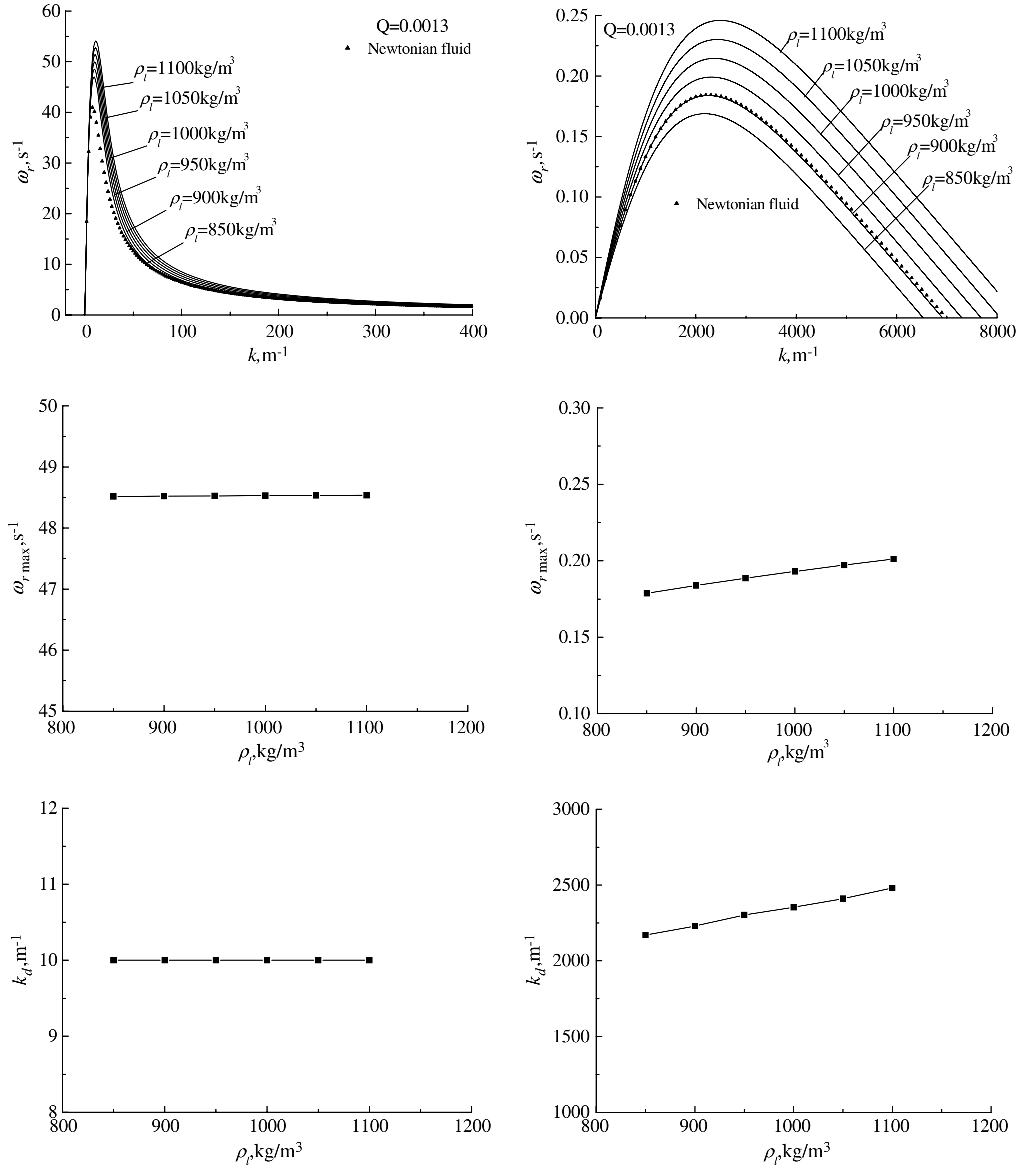


Fig. 10 Effect of liquid densities on the stability of visco-elastic liquid sheet. The left column denotes the sinuous mode; the right column denotes the varicose mode. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s,$U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s, $\rho_l = 910$ kg/m^3 for Newtonian fluid.

Moreover, the breakup time of the visco-elastic liquid sheet is shorter than that of Newtonian liquid sheet for sinuous mode, while for varicose mode they are almost the same, that is, the visco-elastic liquid sheet is more unstable than Newtonian liquid sheet, especially for sinuous mode disturbance.

Apparently, these trends accord with those of Fig. 5.

3. *Effect of the Surface Tension*

The variation of breakup time with the surface tension is plotted in Fig. 13. It is evident that with the increase of the surface tension the

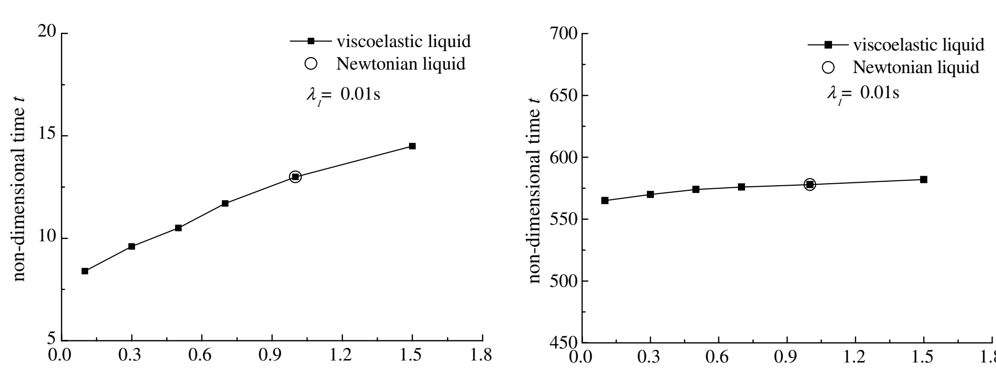


Fig. 11 Variation of the breakup time with the time constant ratio for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\mu = 360$ Pa · s,$U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2$ for Newtonian fluid.

breakup time increases for varicose mode disturbance, but for sinuous mode it is almost constant. Therefore, from Fig. 13 we can get the conclusion that the surface tension can only affect the breakup of liquid sheet driven by varicose mode disturbance by decelerating the breakup, but nearly has nothing to do with that driven by sinuous mode disturbance.

Similarly, the trend that the breakup time of visco-elastic liquid sheet is shorter than that of Newtonian liquid is also observed clearly, which further confirms the conclusion that Newtonian liquid is more stable than visco-elastic liquid.

These conclusions have been gained in Fig. 6.

4. Effect of the Liquid Sheet Velocity

Figure 14 explores the effect of the liquid sheet velocity on the breakup time. With the increase of the liquid sheet velocity the breakup time decreases logarithmically for both sinuous mode and varicose mode disturbance. These mean, the liquid sheet velocity of both visco-elastic and Newtonian liquid will promote the breakup of liquid sheet.

Correspondingly, these curves demonstrate the conclusion that the Newtonian liquid sheet is more stable than visco-elastic liquid sheet, which is particularly evident for the liquid sheet driven by sinuous mode disturbance.

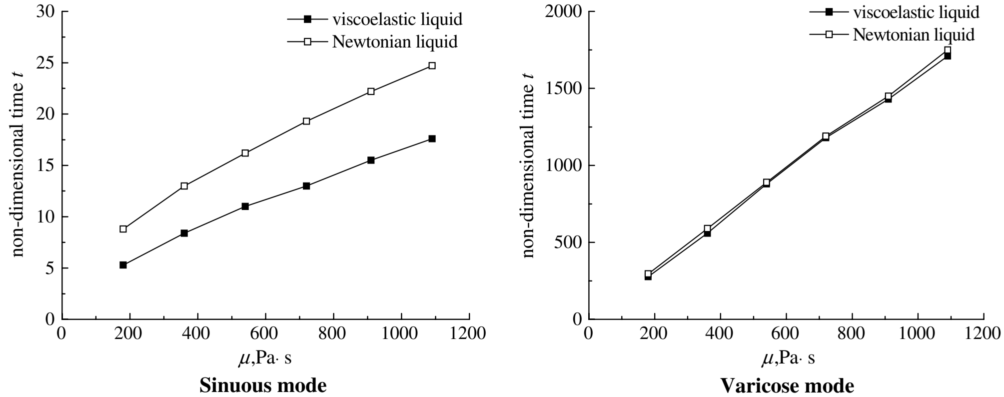


Fig. 12 Variation of the breakup time with the zero shear viscosity (viscosity) for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

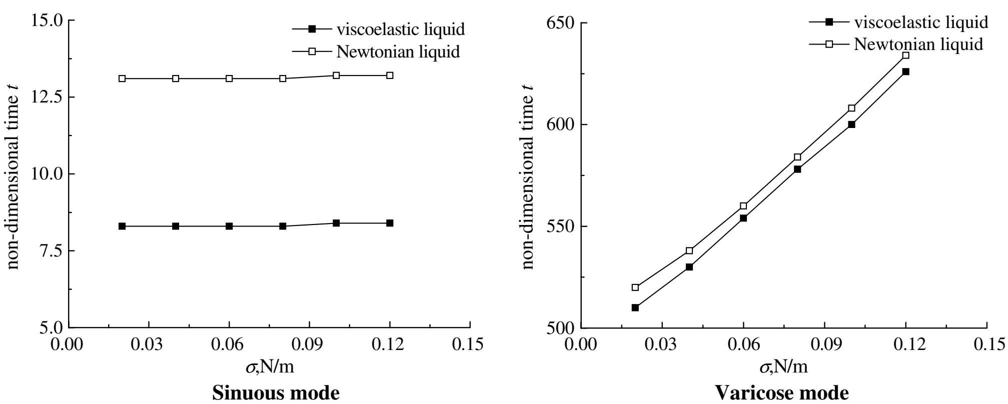


Fig. 13 Variation of the breakup time with the surface tension for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

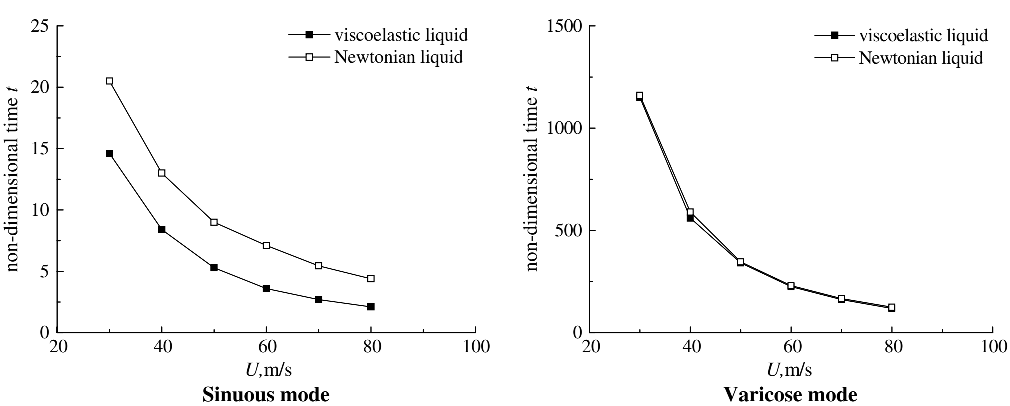


Fig. 14 Variation of the breakup time with the liquid sheet velocity for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

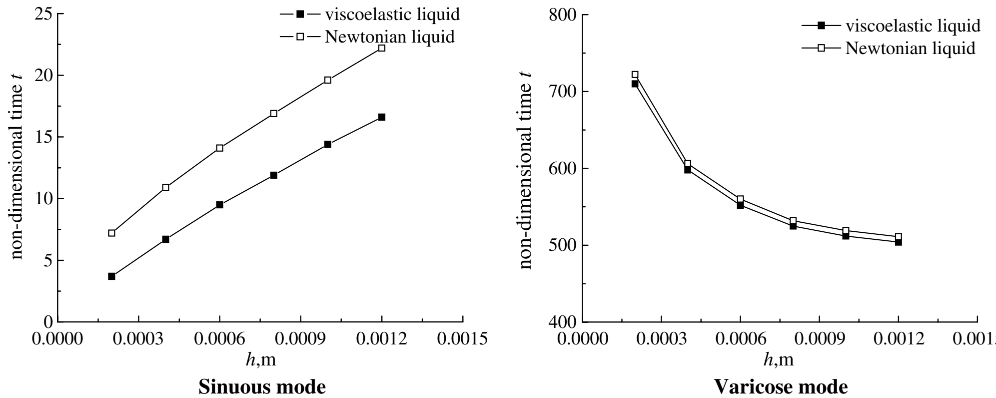


Fig. 15 Variation of the breakup time with the liquid sheet thickness for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $\rho_l = 910$ kg/m^3, $\rho_g = 1.225$ kg/m^3 and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

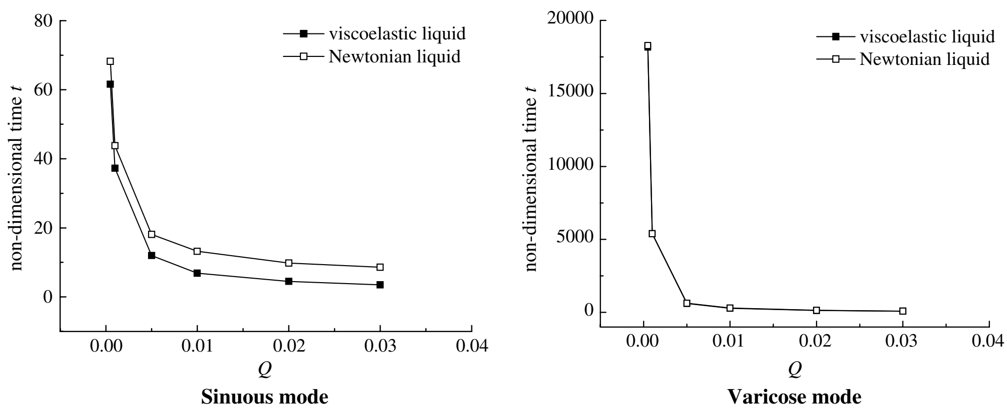


Fig. 16 Variation of the breakup time with the gas-to-liquid density ratio for two modes. For visco-elastic fluid $\lambda_1 = 0.01$ s, $\lambda_2 = 0.001$ s, $\mu = 360$ Pa · s, $U = 20$ m/s, $\sigma = 0.07$ N/m, $h = 0.0005$ m, $\rho_l = 910$ kg/m^3, $Q = \rho_g/\rho_l$ and $\lambda_1 = \lambda_2 = 0$ s for Newtonian fluid.

Distinctly, Fig. 14 and 7 can demonstrate the correctness of each other.

5. Effect of the Liquid Sheet Thickness

Observation of Fig. 15 reveals that when the liquid sheet is thicker it is more difficult to break up for the liquid sheet driven by sinuous mode disturbance, but easier for varicose mode, which is represented by the increase or decrease of the breakup time.

Also, the breakup time of the Newtonian liquid sheet is longer than that of the visco-elastic liquid sheet for two modes disturbance and sinuous mode disturbance in particular.

These conclusions can be proved by combining Fig. 15 and 8.

6. Effect of the Gas-to-Liquid Density Ratio

The effect of the gas-to-liquid density ratio on the breakup time of liquid sheet is examined in Fig. 16 for sinuous and varicose disturbances, respectively. These curves denote that the breakup time of the liquid sheet decreases with the increase of the gas-to-liquid density ratio for both sinuous and varicose modes disturbance. However, it does not decrease linearly but logarithmically. Concretely, when the gas-to-liquid density ratio is small (under 0.005) the breakup time decreases sharply, while when the gas-to-liquid density ratio is great that decreases gently. Therefore, it is acceptable to get the conclusion that a greater gas-to-liquid density ratio can expedite the breakup process of the liquid sheet.

By comparing the visco-elastic liquid and Newtonian liquid, it is obviously to find that the breakup time of visco-elastic liquid is shorter than that of Newtonian liquid for sinuous mode disturbance, but they are almost as same as each other for varicose mode disturbance. Thus, the Newtonian liquid sheet is more stable than visco-elastic liquid sheet for sinuous mode.

These results can also be seen in Fig. 9.

IV. Conclusions

The instability of visco-elastic liquid sheets moving in an inviscid gaseous environment is investigated from two aspects in this paper: the disturbance growth rate and the breakup time. Moreover, the Newtonian liquid is also investigated with the purpose of comparing. Based on these above analysis, the conclusions can be drawn as following:

1) With the same liquid property (except λ_1 and λ_2), the disturbance growth rate of visco-elastic liquid sheet is higher than that of Newtonian liquid sheet for sinuous mode disturbance, and almost the same for varicose mode disturbance. Accordingly, the breakup time of the Newtonian liquid sheet is longer than that of the visco-elastic liquid sheet for sinuous mode and almost the same for varicose mode disturbance, indicating that visco-elastic liquid sheet is more unstable than Newtonian liquid sheet.

2) The maximum growth rate of sinuous mode disturbance is far greater than that of varicose mode, while the opposite is true for the dominant wave number. And the breakup time of the liquid sheet driven by varicose mode disturbance is much longer than that of sinuous mode disturbance. These mean, the sinuous mode disturbance is more unstable than varicose mode, or sinuous mode disturbances always prevail over varicose mode disturbances for visco-elastic liquid sheets. Moreover, the sinuous mode disturbance wave in the breakup process would present a long wave as its dominant wave number is much less than that of the varicose mode disturbance.

3) The maximum growth rate and corresponding dominant wave number of sinuous mode disturbance decrease with the increase of the time constant ratio, liquid sheet thickness and the zero shear

viscosity, while they increase with the increase of the liquid velocity and the gas-to-liquid density ratio, indicating that the former three parameters may resist the instability of visco-elastic liquid sheet for sinuous mode disturbance but the latter two parameters enhance it. However, the surface tension can hardly affect the growth rate of sinuous mode disturbance.

4) When the liquid sheet velocity, liquid sheet thickness and gas-to-liquid density ratio are increased, the maximum growth rate and corresponding dominant wave number increase for varicose mode disturbance, while the opposite is true for zero shear viscosity and surface tension. This denotes that the former three parameters can enhance the instability behavior of visco-elastic liquid sheet for varicose mode disturbance, while the latter two parameters weaken it. Nevertheless, the time constant ratio has noting to do with the growth rate of varicose mode disturbance almost. These trends of (3) and (4) can also be verified by the breakup time curves.

5) When the gas-to-liquid density ratio is constant, the liquid density can hardly affects the growth rate for sinuous mode and affects slightly for varicose mode. Thus, it is concluded that for sinuous mode disturbance the effects of gas and liquid densities on the growth rate is induced by the gas-to-liquid density ratio Q directly, and for varicose mode when Q is constant the liquid density can enhance the instability to a certain extent.

6) The instability range of varicose mode disturbance increases with the increase of the liquid velocity and the gas-to-liquid density ratio, but decreases with the increase of the surface tension. And when the wave number is great the effects of all the parameters on the growth rate is slight for sinuous mode disturbance.

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